Lecture 7

Decomposing consumer responses into Income and Substitution effects.

When the price of a good changes relative to another, two things happen:

1) One good becomes relatively more expensive, and the other good relatively less.
2) The total purchasing power of a consumer's income changes.

The substitution effect refers to the change in consumption choices resulting from a change in relative prices.

→ It always works in the opposite direction i.e. when the price of one good relative to another rises, consumption of the former falls, and vice versa.

The Income effect refers to the change in consumption choices resulting from a change in purchasing power.

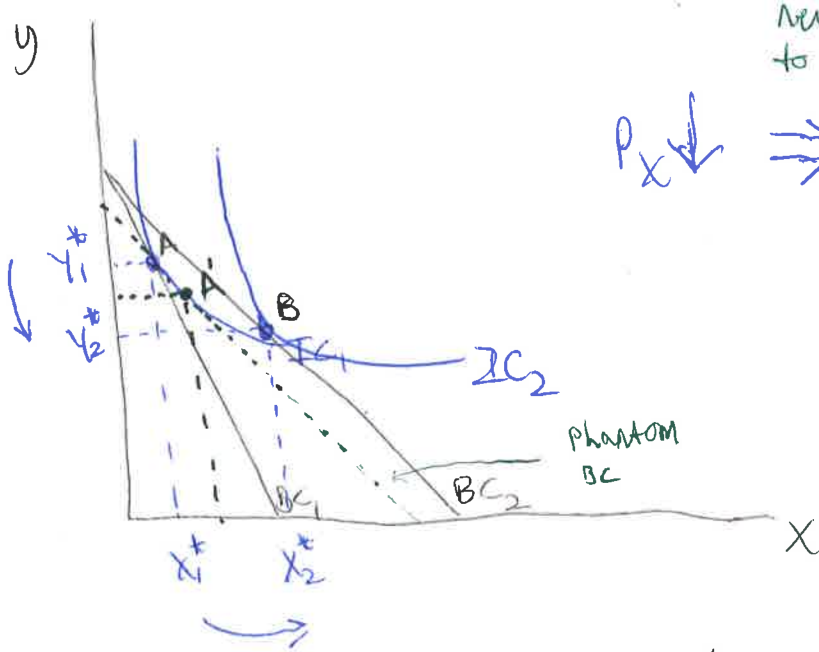
↳ This can be negative or positive (inferior or normal goods)

i.e. $\frac{dx}{dm} > 0 \Rightarrow$ Normal

$\frac{dx}{dm} < 0 \Rightarrow$ inferior

Hicks Decomposition: Draw a "phantom" Budget constraint parallel to the new budget constraint and tangent to the old indifference curve

$P_x \downarrow \Rightarrow Y^* \downarrow, X^* \uparrow$



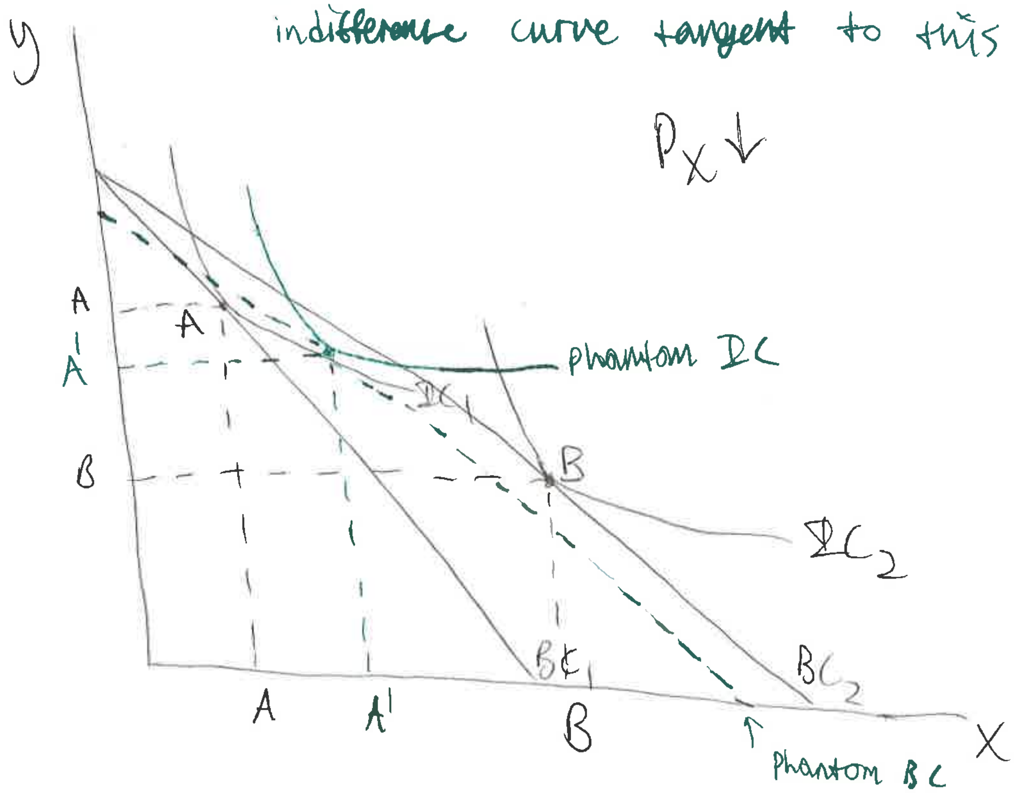
Note that the income effect here is quite large because the consumer was initially spending a large share of income on y, and y became relatively more expensive.

- From A to A' represents the substitution effect. A to A' represents a substitution away from y and into x (because y became relatively more expensive).

- From A' to B represents the income effect. It is positive here for good x, so good x must be normal. The income effect is negative here for good y (so good y must be inferior)

Slutsky Decomposition: Draw a "phantom" Budget constraint parallel to the new B.C. Draw a "Phantom" indifference curve tangent to this "phantom" BC.

$P_X \downarrow$

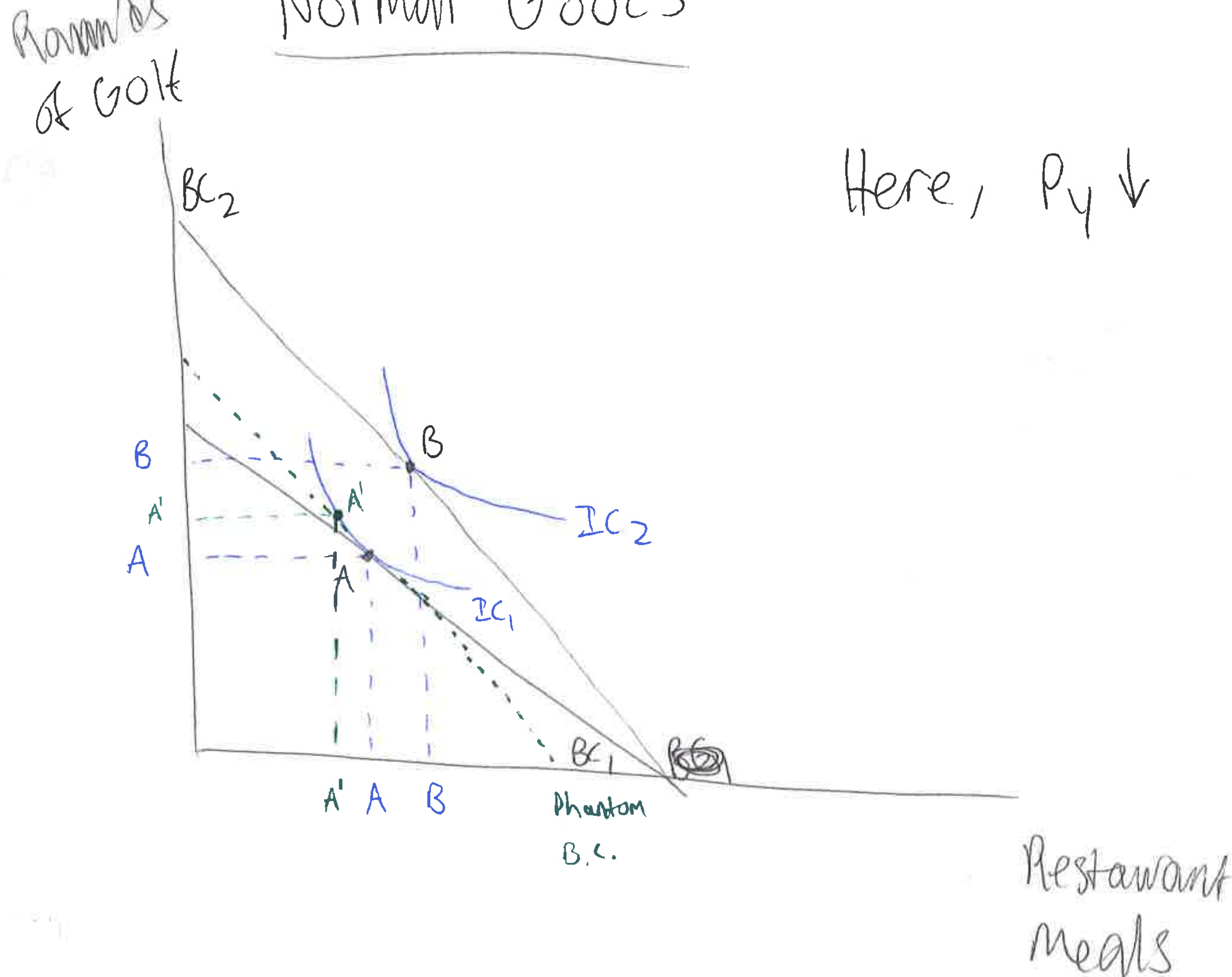


- From A to A' represents the substitution effect. Here, the consumer substitutes away from good y and into good x (because y has become relatively more expensive).

- From A' to B represents the income effect. Here, within the income effect, the consumer consumes less good y and more good x. Thus, good y must be inferior and good x normal

Normal Goods

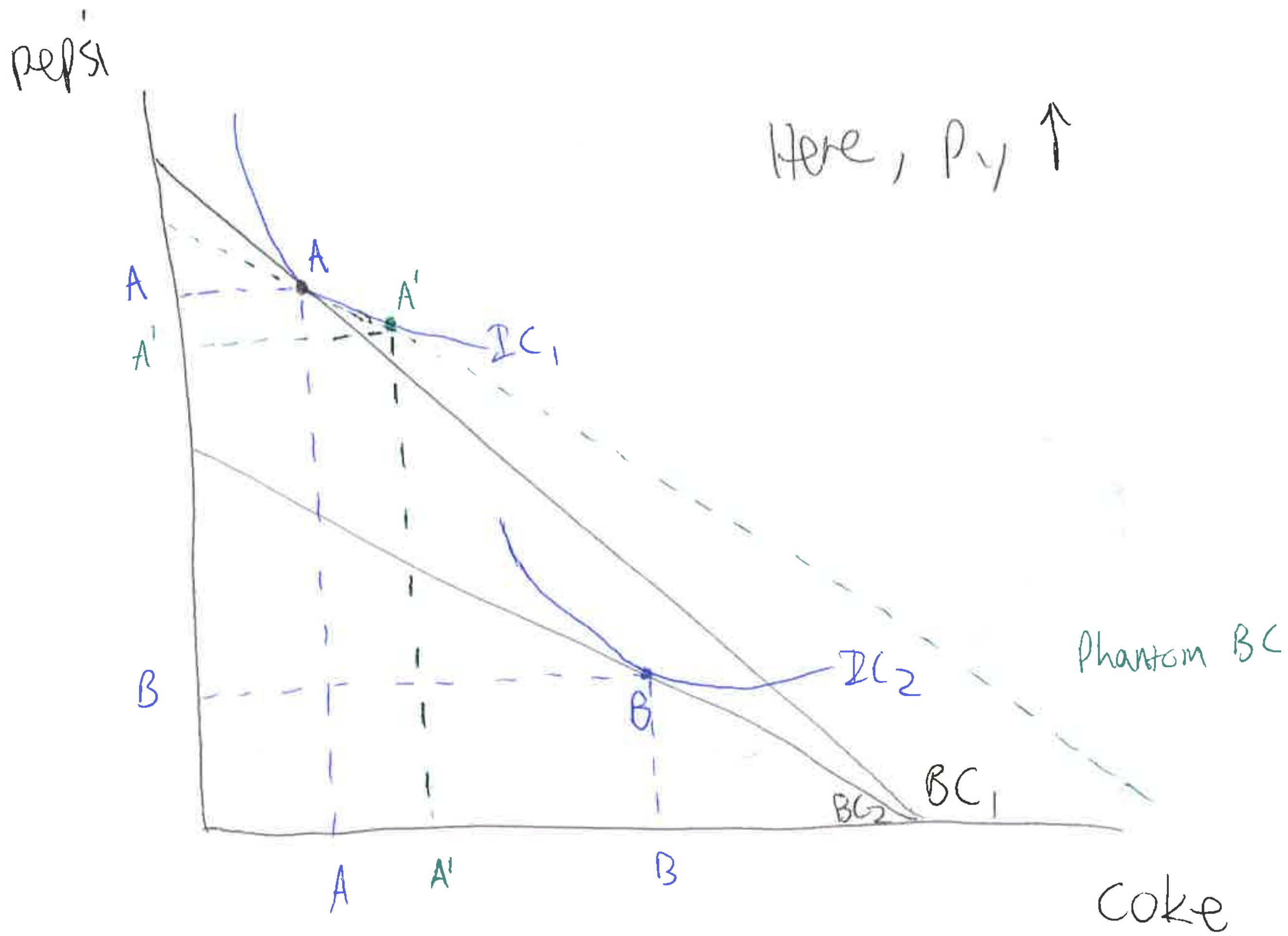
Here, $P_y \downarrow$



- From A to A' represents the substitution effect. Notice that here, the consumer substitutes away from x and into y, this is because y has become relatively less expensive.

- From A' to B represents the income effect, and it is positive for both goods, indicating that both goods are normal goods. Note that the positive ...come effect for good x outweighs the negative substitution effect.

A price increase



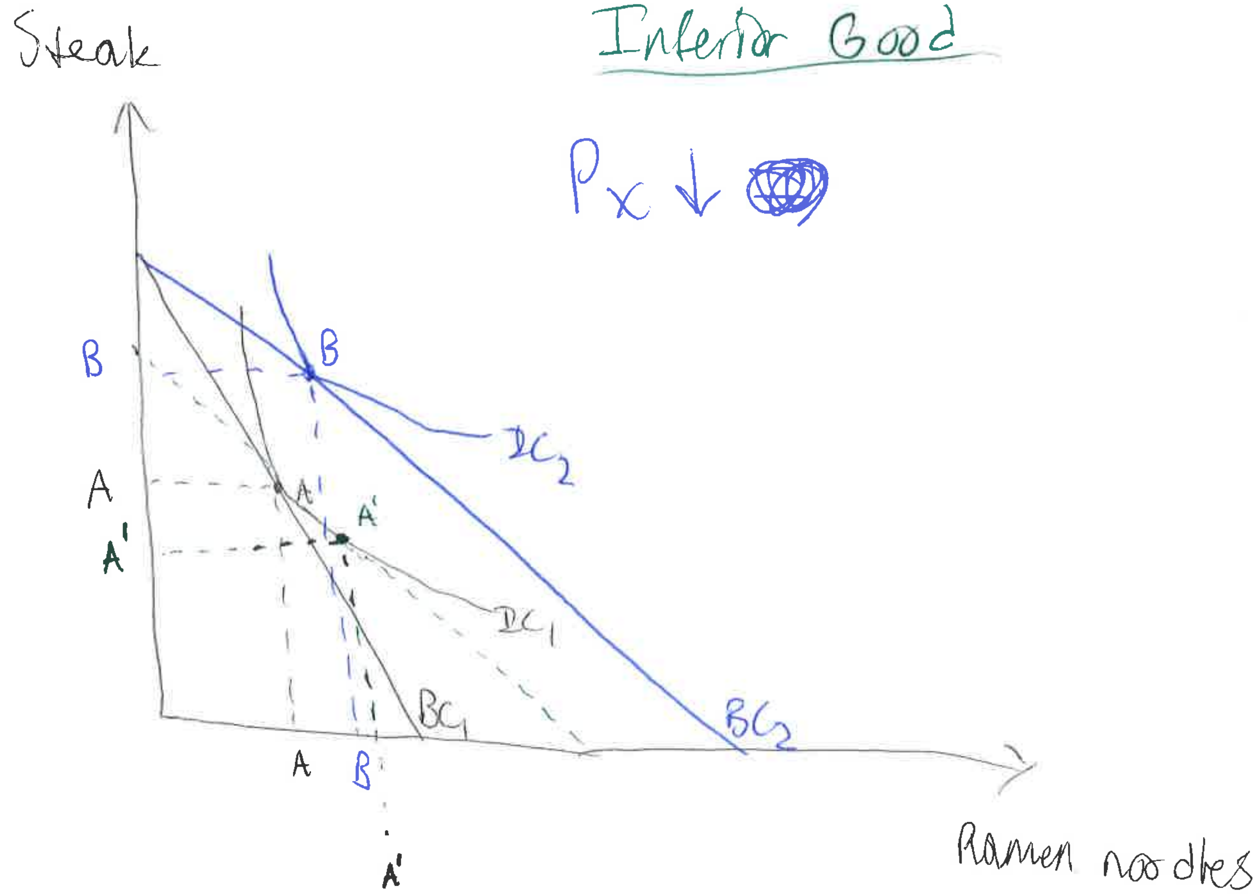
• From A to A' represents the substitution effect. Here, the consumer substitutes away from pepsi and into coke, because pepsi became relatively more expensive

• From A' to B represents the income effect. Here there is a large negative income effect decreasing consumption of pepsi (Note how much of the consumer's budget was initially devoted to pepsi, quite a large share). There is a large positive income effect increasing consumption of coke.

Note: $P_y \uparrow \Rightarrow y \downarrow, x \uparrow$, Here purchasing power decreased with respect to pepsi, as did Y^*.

Inferior Good

$P_x \downarrow$



- From A + A' represents the substitution effect. Here, the price of X decreases, so the consumer substitutes away from steak and into Ramen.

- From A' to B represents the income effect. Here, the consumer eats more steak and less ramen within the income effect, so steak must be normal and Ramen inferior. Notice that for Ramen, the subs. and income effects work in opposite directions, however the subs. effect outweighs the income effect. So the consumer still demands more Ramen despite it being an inferior good.

Giffen goods: are goods for which quantity demanded increases as the price rises.

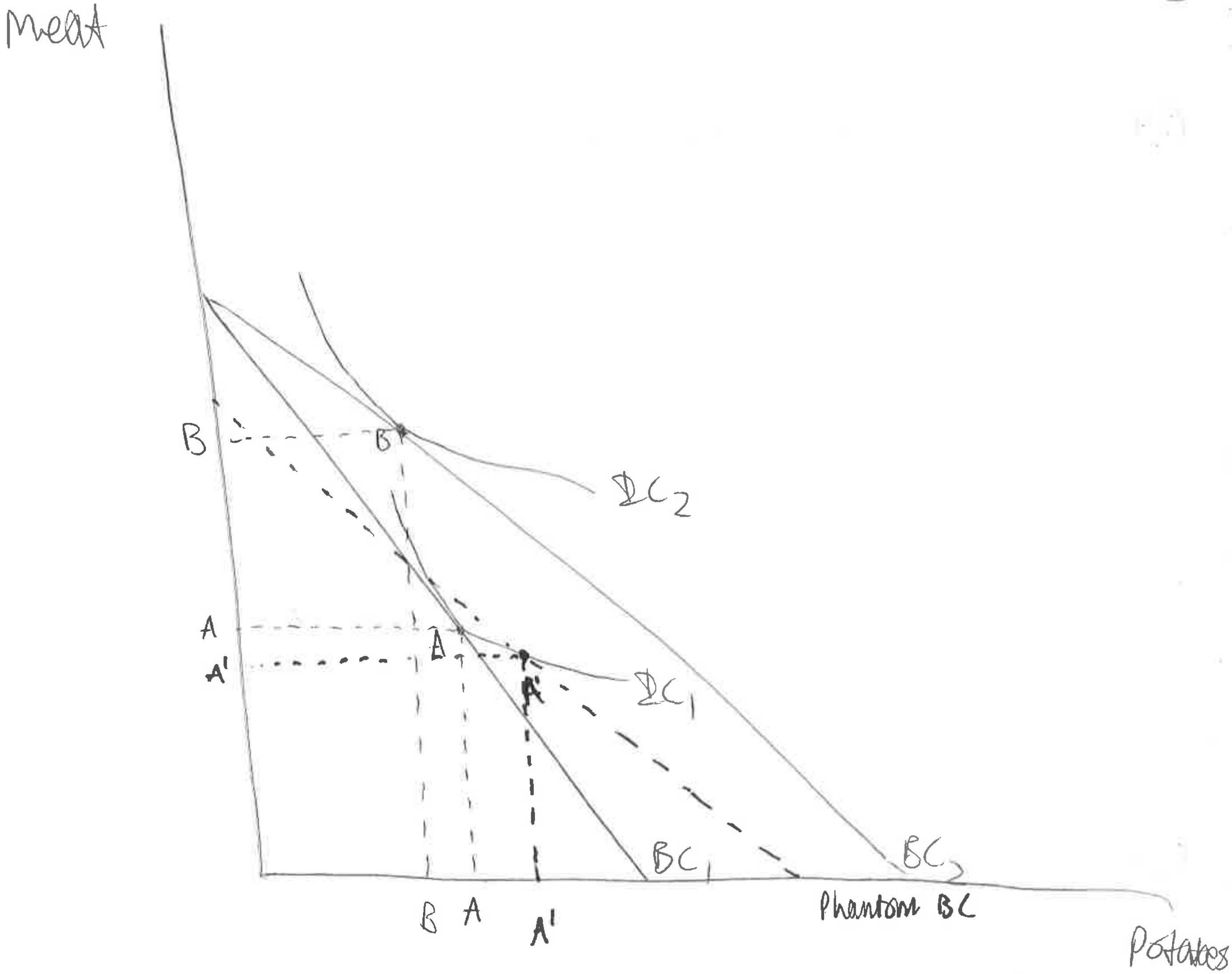
→ Inferior good but the income outweighs the substitution effect

→ Results in an upward sloping demand curve.

When the price of a giffen good drops, the substitution effect (which acts to increase demand) is smaller than the income effect.

• Economists struggle to find any real world examples of a giffen good, however it is theoretically possible.

An often discussed example are potatoes during the Irish potatoe famine. During the mid-1800s there was a severe famine of potatoes, a staple of the Irish diet at the time. This drastically reduced the purchasing power of consumers, shrinking the bundles of goods that they could afford. The resulting income effect led to a decrease in the demand for other foods such as meat that were normal goods and an increase in the demand for potatoes, an inferior good.



- Here from A to A' represents the substitution effect. Consumers are substituting away from meat and into potatoes.

- Here from A' to B represents the income effect. The income effect is positive for meat, indicating that meat is a normal good. The income effect is negative for potatoes, indicating that it is an inferior good. The income effect dominates the substitution effect for potatoes (which leads to a decrease in the demand for potatoes despite a drop in the price of potatoes) ⇒ potatoes is a special type of inferior good, a giffen good.

- Lets derive an analytical representation of a decomposition of a consumer's response to a price change. First, let's return to the most important model of consumer theory, the utility maximization model. Assume a Cobb-Douglas utility function, but leave the Budget Constraint in a general form.

The Consumer's utility maximization problem is:

$$\begin{cases} \max\limits_{\{x,y\}} & u(x,y) = x^{\alpha} y^{\beta} \\ \text{s.t.} & M = P_x X + P_y Y \end{cases}$$

This is called the "primal" problem.

Lets solve using the tangency condition:

$$MRS_{xy} = \frac{P_x}{P_y} \qquad MRS_{xy} = \frac{MU_x}{MU_y} = \frac{\alpha x^{\alpha-1} y^{\beta}}{\beta x^{\alpha} y^{\beta-1}} = \frac{\alpha}{\beta} \cdot \frac{Y}{X}$$

$$\hookrightarrow \frac{\alpha}{\beta} \frac{Y}{X} = \frac{P_x}{P_y}$$

$$\Rightarrow Y = \frac{\beta}{\alpha} \frac{P_x}{P_y} X$$

Plug this into the constraint:

P_y's cancel here because of our Cobb-Douglas functional form

$$M = P_x X + \cancel{P_y}\left(\frac{\beta}{\alpha} \frac{P_x}{\cancel{P_y}} X\right)$$

$$M = P_x X + \frac{\beta}{\alpha} P_x X$$

$$= \left(1 + \frac{\beta}{\alpha}\right) P_x X = \left(\frac{\alpha}{\alpha} + \frac{\beta}{\alpha}\right) P_x X \Rightarrow \boxed{X = \frac{\alpha}{\alpha+\beta} \frac{M}{P_x}}$$

$1 = \frac{\alpha}{\alpha}$

by symmetry,

$$\boxed{Y = \frac{\beta}{\alpha+\beta} \frac{M}{P_y}}$$

These are Marshallian demand functions

Notice what the Marshallion demand functions are a function of (what inputs do they depend on?). They depend on prices P_x and P_y and income M. Hence we can write the Marshallion demand function as $X^M(P_x, P_y, m)$

Marshallian.

Marshallian demand functions are also called "uncompensated demand" as we are holding income as fixed, i.e we do not compensate the consumer with more income.

Now, let's use the same Cobb-Douglas utility and let's derive the solution to the "Dual" problem, or the consumer's expenditure minimization problem.

The consumer's expenditure minimization problem is:

$$\begin{cases} \min\limits_{\{x,y\}} & E = P_x X + P_y Y \\ \text{s.t.} & \bar{u} = X^{\alpha} Y^{\beta} \end{cases}$$

This is called the "Dual" problem.

We can solve this using the same tangency condition as before. In the previous problem, we established a relationship between x and y of the form:

$$Y = \frac{\beta}{\alpha}\frac{P_x}{P_y} X$$

once again, plug this into the constraint (this time a constraint on utility).

$$\bar{u} = X^{\alpha}\left[\frac{\beta}{\alpha}\frac{P_x}{P_y} X\right]^{\beta} = \left[\frac{\beta}{\alpha}\frac{P_x}{P_y}\right]^{\beta} X^{\alpha+\beta}$$

$$\Rightarrow X^{\alpha+\beta} = \left[\frac{\alpha}{\beta}\frac{P_y}{P_x}\right]^{\beta} \bar{u}$$

$$X = \left[\frac{\alpha}{\beta}\frac{P_y}{P_x}\right]^{\frac{\beta}{\alpha+\beta}} \bar{u}^{1/\alpha+\beta}$$ by symmetry, $$Y = \left[\frac{\beta}{\alpha}\frac{P_x}{P_y}\right]^{\frac{\alpha}{\alpha+\beta}} \bar{u}^{\frac{1}{\alpha+\beta}}$$

These are Hicksian demand functions

Notice what the Hicksian demand functions are a function of (what inputs do they depend on?). They depend on prices, P_x and P_y, and some fixed level of utility $\bar{u}$. Hence we can write the Hicksian demand functions as

$$X^H(P_x, P_y, \bar{u})$$

Hicksian.

Hicksian demand functions are also called "compensated" demand as in order to keep the consumer ~~at~~ the same indifference curve, the consumer is given additional income.

- Hence we have shown $X^M(P_x, P_y, M)$ and $X^H(P_x, P_y, \bar{u})$ where do these two demand functions equal one another? It occurs at the point in which $\bar{u}$ is the max utility and M is the min expenditure, i.e. $M = E(P_x, P_y, \bar{u})$, where E represents the expenditure function and $\bar{u}$ represents the max level of utility. Equating the two demand functions we have

$$X^M(P_x, P_y, M) = X^H(P_x, P_y, \bar{u})$$

substituting the expenditure function in for income:

$$x^M(P_x, P_y, E(P_x, P_y, \bar{u})) \equiv x^H(P_x, P_y, \bar{u})$$

This is known as the Slutsky identity.

Let's differentiate both sides with respect to P_x:

$$\underbrace{\frac{dx^M}{dP_x}}_{\text{direct effect}} + \underbrace{\frac{dx^M}{dM}}_{\text{indirect effect}} \cdot \frac{\partial E}{\partial P_x} = \frac{dx^H}{dP_x}$$

There is an identity known as Sheppard's Lemma that tells us $\frac{\partial E}{\partial P_x} = x^H$

Rearranging the above expression:

$$\boxed{\frac{dx^M}{dP_x} = \underbrace{\frac{dx^H}{dP_x}}_{\text{substitution effect}} - \underbrace{\frac{dx^M}{dM} \cdot x^H}_{\text{income effect}}}$$

This is a famous result in economics called the Slutsky equation

This is an analytical derivation of the graphical results we saw earlier. Note, the substitution effect is always negative, and the income effect can be either positive or negative (depending on whether the good is normal or inferior).

The Marshallian demand function is derived from the consumer's utility maximization problem. Assuming Cobb-Douglas utility, the consumer's problem is:

$$\begin{cases} \max\limits_{\{x,y\}} & u(x,y) = x^{\alpha} y^{\beta} \\ \text{s.t.} & M = P_x X + P_y Y \end{cases}$$

Let's solve this problem using the Lagrangian method. Our Lagrangian function is:

$$\mathcal{L} = x^{\alpha} y^{\beta} + \lambda (M - P_x X - P_y Y)$$

<u>First-order conditions:</u>

$$\left.\begin{aligned} [x]&: \alpha x^{\alpha-1} y^{\beta} - \lambda P_x = 0 \\ [y]&: \beta x^{\alpha} y^{\beta-1} - \lambda P_y = 0 \\ [\lambda]&: M - P_x X - P_y Y = 0 \end{aligned}\right\}$$

This is a system of 3 equations and 3 unknowns (x, y, λ)

Dividing the first 2 FOCs:

$$\frac{\alpha x^{\alpha-1} y^{\beta}}{\beta x^{\alpha} y^{\beta-1}} = \frac{\cancel{\lambda} P_x}{\cancel{\lambda} P_y}$$

$$\frac{\alpha y}{\beta x} = \frac{P_x}{P_y} \implies y = \frac{\beta}{\alpha} \frac{P_x}{P_y} X$$

Plugging this expression into the constraint gives

$$M = P_X X + \cancel{P_Y}\left(\frac{\beta}{\alpha}\frac{P_X}{\cancel{P_Y}}X\right)$$

$$M = P_X X + \frac{\beta}{\alpha} P_X X$$

$$M = P_X X\left(1 + \frac{\beta}{\alpha}\right)$$

$$M = P_X X\left(\frac{\alpha}{\alpha} + \frac{\beta}{\alpha}\right)$$

$$M = P_X X\left(\frac{\alpha+\beta}{\alpha}\right)$$

$$\Rightarrow X = \frac{\alpha}{\alpha+\beta}\frac{M}{P_X}$$

by symmetry,

$$\Rightarrow Y = \frac{\beta}{\alpha+\beta}\frac{M}{P_Y}$$

These are marshallian demand functions

Notice what they depend on. (P_X, P_Y, and M)

i.e. $X^*_m(P_X, P_Y, M)$ and $Y^*_m(P_X, P_Y, M)$

where this m stands for marshallian.

The marshallian demand function specifies what the consumer would buy in each price and income situation. These are also called uncompensated demand functions.

The Hicksian demand function is derived from the consumer's expenditure minimization problem.

Assuming Cobb-Douglas utility, the consumer's problem is:

$$\begin{cases} \min\limits_{\{x,y\}} \; E = P_X X + P_Y Y \\ \text{s.t. } X^\alpha Y^\beta = \bar{u} \end{cases}$$

Let's solve this problem using the Lagrangian method.

Our Lagrangian function is:

$$\mathcal{L} = P_X X + P_Y Y + \lambda(\bar{u} - X^\alpha Y^\beta)$$

<u>First-order conditions:</u>

$$[x]: \; P_X - \lambda \cdot \alpha X^{\alpha-1} Y^\beta = 0$$

$$[y]: \; P_Y - \lambda \cdot \beta X^\alpha Y^{\beta-1} = 0$$

$$[\lambda]: \; \bar{u} - X^\alpha Y^\beta = 0$$

This is a system of 3 equations and 3 unknowns (X, Y, λ)

Dividing the first 2 FOCs:

$$\frac{P_X}{P_Y} = \frac{\cancel{\lambda}\alpha X^{\alpha-1} Y^\beta}{\cancel{\lambda}\beta X^\alpha Y^{\beta-1}} \Longrightarrow \frac{P_X}{P_Y} = \frac{\alpha}{\beta}\frac{Y}{X}$$

So $Y = \frac{\beta}{\alpha}\frac{P_X}{P_Y} X$

Plugging this expression into the constraint gives

$$\bar{u} = x^{\alpha} y^{\beta}$$

$$\bar{u} = x^{\alpha}\left(\frac{\beta}{\alpha}\frac{P_x}{P_y}x\right)^{\beta}$$

$$\bar{u} = x^{\alpha+\beta}\left(\frac{\beta}{\alpha}\frac{P_x}{P_y}\right)^{\beta}$$

$$\Rightarrow x^{\alpha+\beta} = \bar{u}\left(\frac{\beta}{\alpha}\frac{P_x}{P_y}\right)^{-\beta}$$

$$x = \left(\frac{\beta}{\alpha}\frac{P_x}{P_y}\right)^{\frac{-\beta}{\alpha+\beta}} \bar{u}^{\frac{1}{\alpha+\beta}}$$

$$\Rightarrow x = \left(\frac{\alpha}{\beta}\frac{P_y}{P_x}\right)^{\frac{\beta}{\alpha+\beta}} \bar{u}^{\frac{1}{\alpha+\beta}}$$

by symmetry,

$$\Rightarrow y = \left(\frac{\beta}{\alpha}\frac{P_x}{P_y}\right)^{\frac{\alpha}{\alpha+\beta}} \bar{u}^{\frac{1}{\alpha+\beta}}$$

These are Hicksian demand functions

Notice what they depend on (P_x, P_y, and $\bar{u}$)

i.e. $x^*_H(P_x, P_y, \bar{u})$ and $y^*_H(P_x, P_y, \bar{u})$

where this H stands for Hicksian

The Hicksian demand function specifies what the consumer would buy while minimizing expenditure and maintaining a fixed level of utility. These are also called compensated demand functions.